Analysis of RO data retrieved from the Wigner distribution function

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Reference: M. E. Gorbunov, K. B. Lauritsen, S. S. Leroy:
Wigner distribution function as a time-frequency joint representation of radio occultation signals,
Radio Science, Vol. 45, RS6011, doi:10.1029/2010RS004388 (2010);

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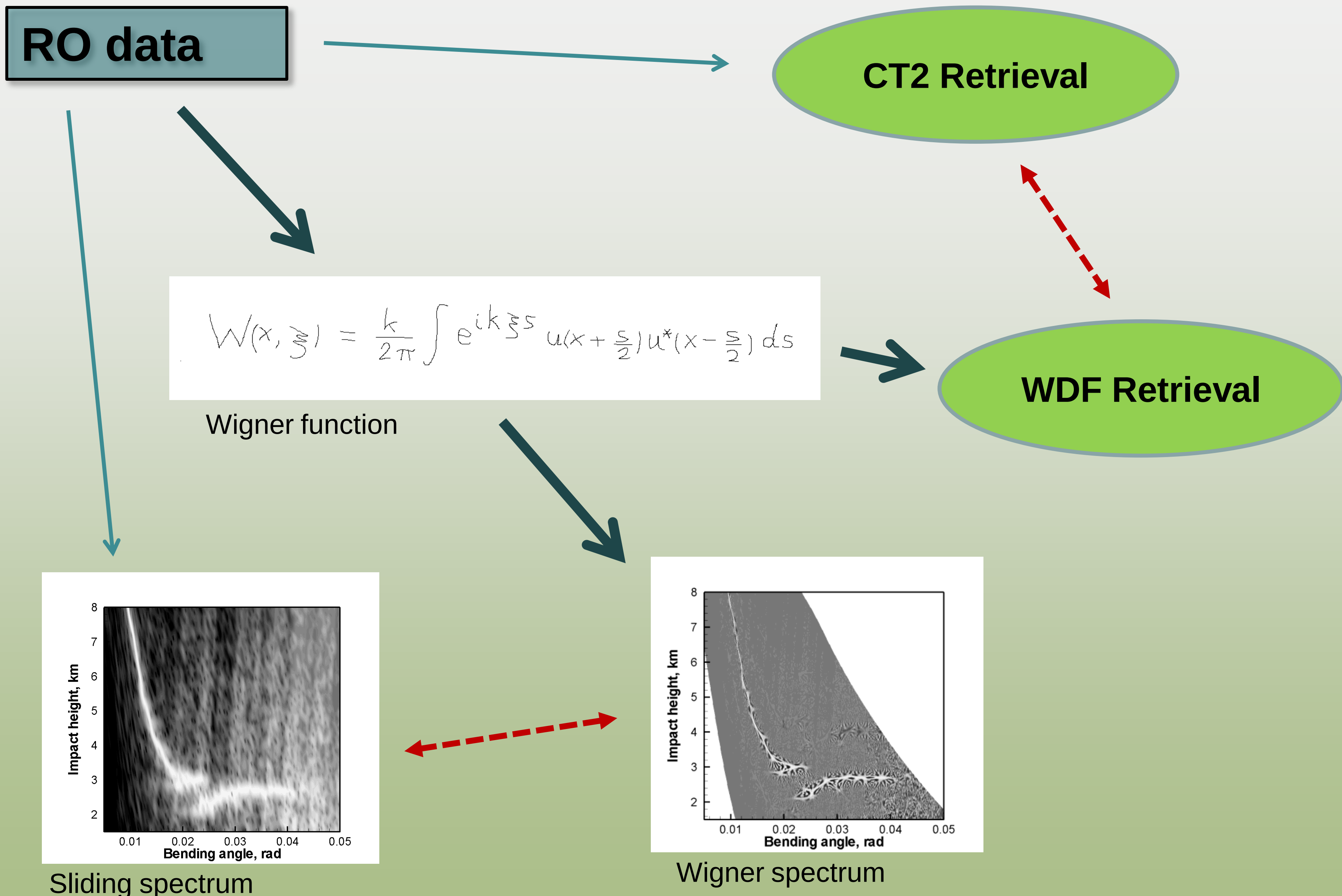
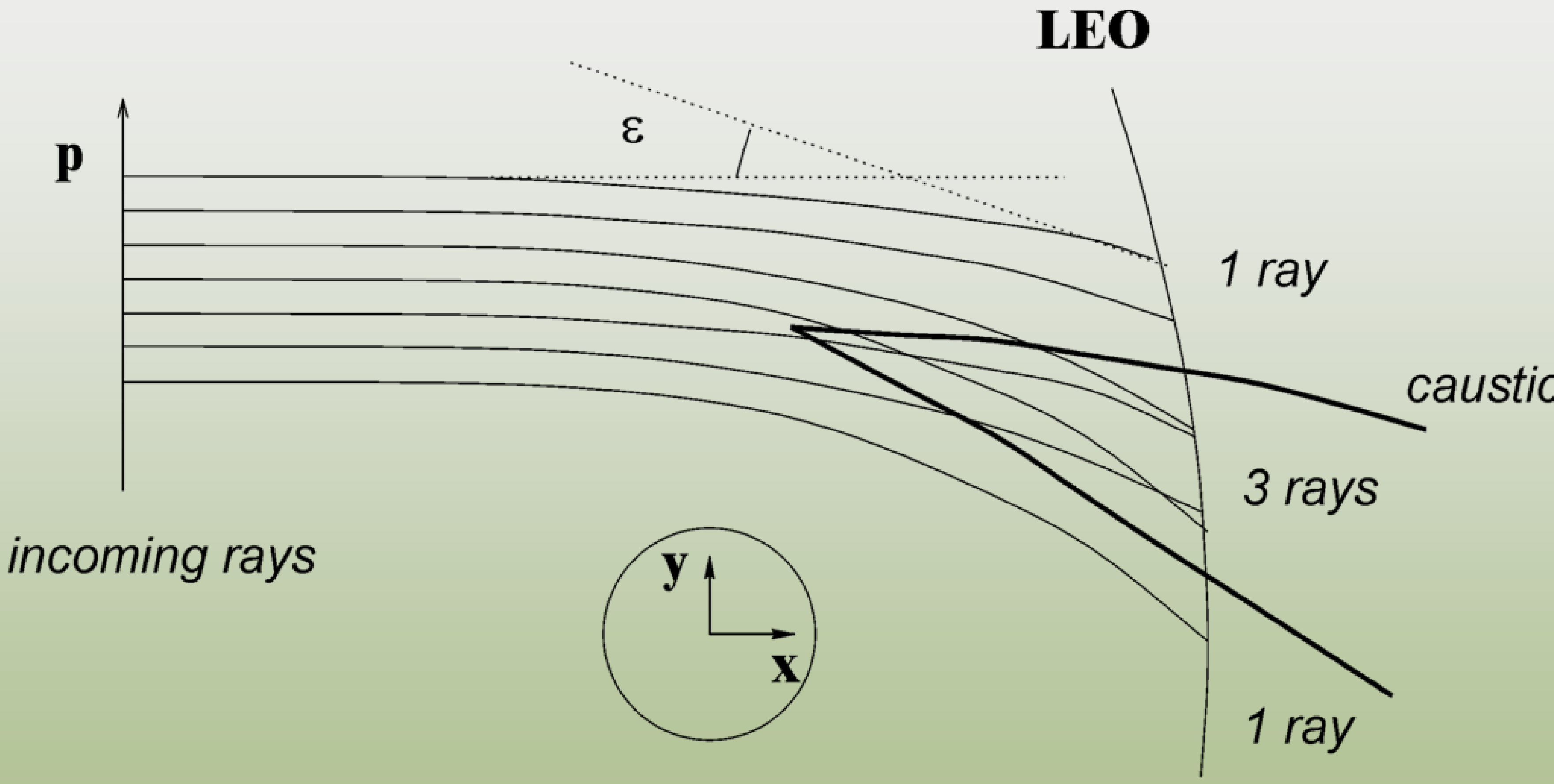
ROM SAF


RO data
CT2 Retrieval
$W(x,\xi) = \frac{k}{2\pi}\int e^{ik\xi s}\, u(x+\frac{s}{2})\, u^*(x-\frac{s}{2})\, ds$
Wigner function
WDF Retrieval
Impact height, km
Bending angle, rad
0.01 0.02 0.03 0.04 0.05
2 3 4 5 6 7 8
Sliding spectrum
Wigner spectrum

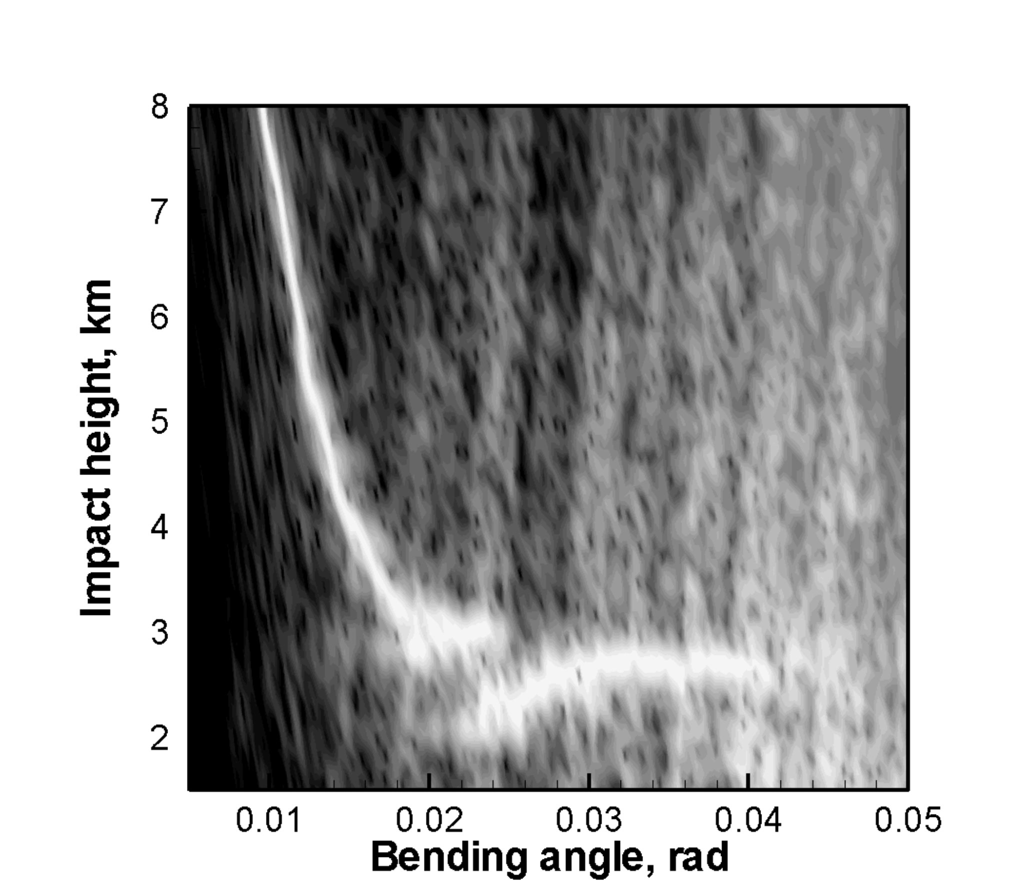
Multipath example



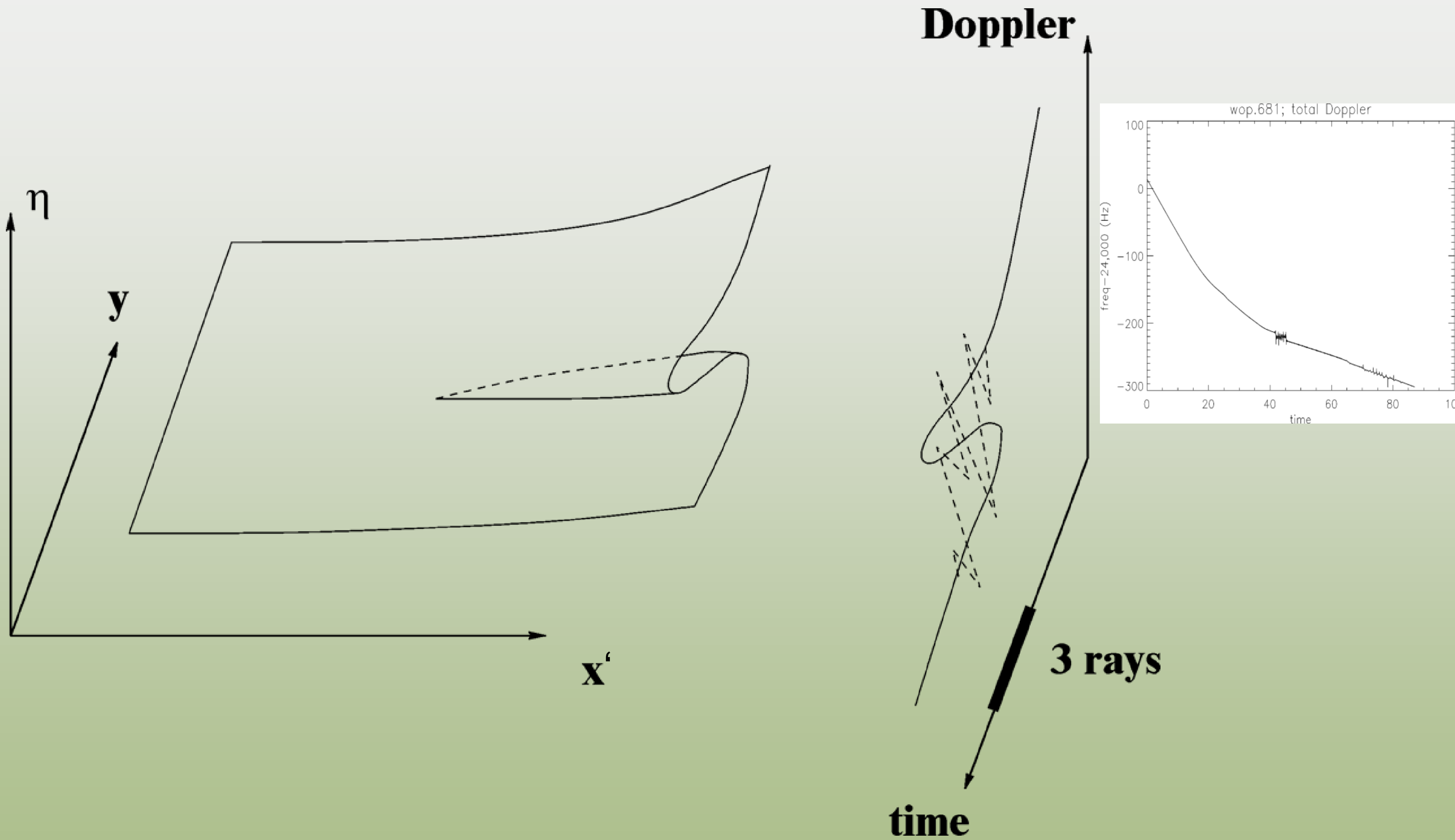
Sliding Fourier spectrum of *u*(*t*)

2D sliding Fourier spectrum of *u*(*t*) mapped to (BA,IP) space

Example: COSMIC data with sharp BA structure at the PBL



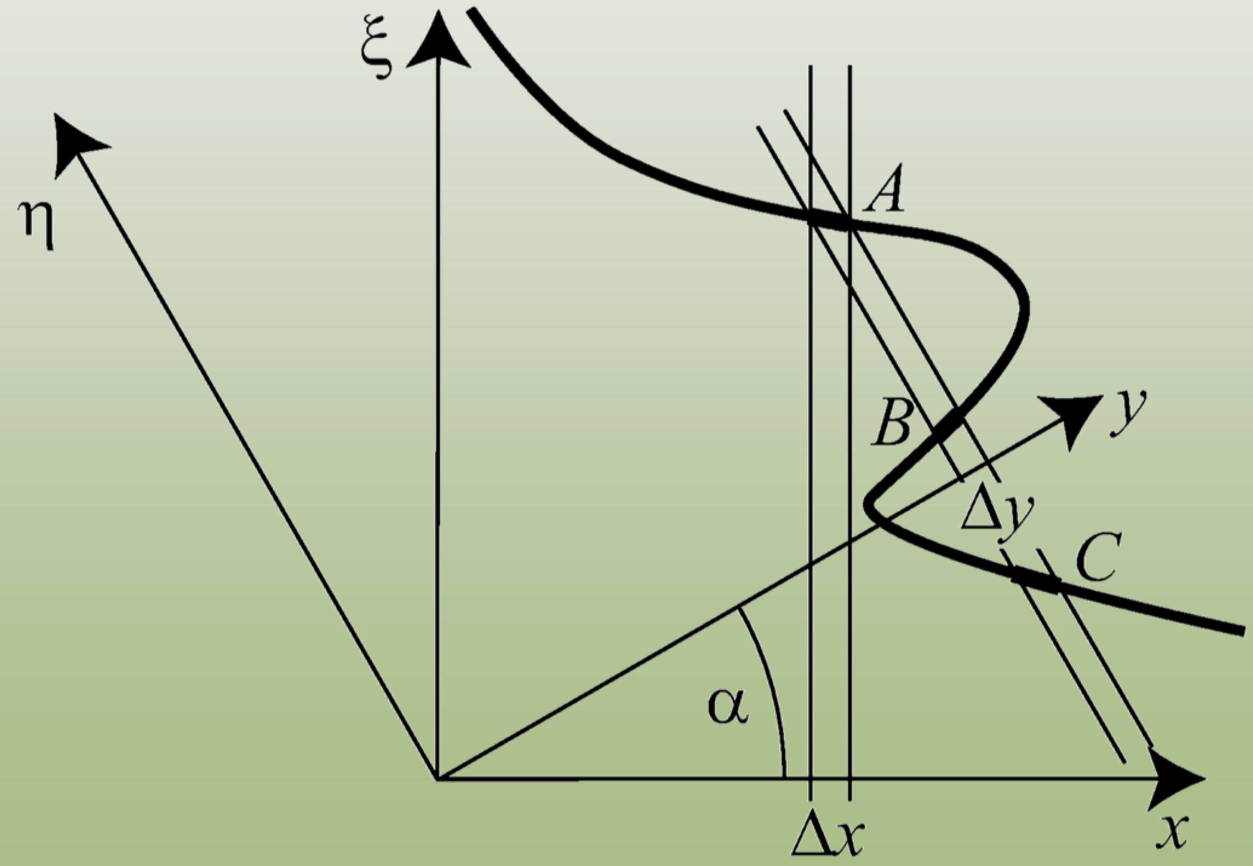
Schematic ray manifold



Basic idea

Rotations in phase space by angle α

Different projections of ray manifold



Definition of $W(x,\xi)$

Energy density constraint from tomographic projections:

$$\iint W(x,\xi)\,\delta(x\cos\alpha + \xi\sin\alpha - y)\,dx\,d\xi = \left|\hat{\Phi}_\alpha[u](y)\right|^2$$

FIO operator rotating $u(t)$ the angle α

Analytical solution:

$$W(x,\xi) = \frac{k}{2\pi}\int e^{ik\xi s}\,u\left(x+\frac{s}{2}\right)u^*\left(x-\frac{s}{2}\right)ds$$

Solution coincides with the Wigner distribution function introduced by E. P. Wigner in 1932 in quantum mechanics;

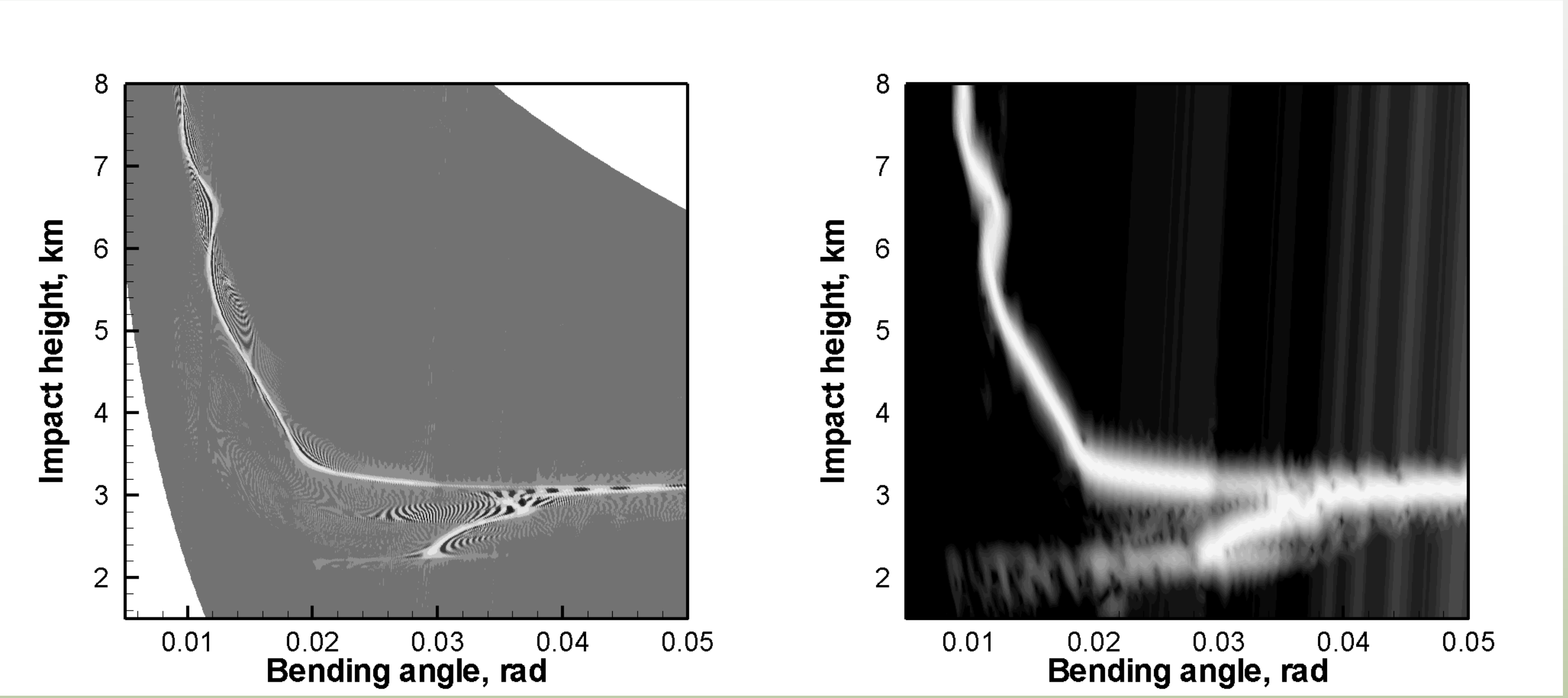
Previous applications of WDF and other time-frequency joint representations: Ville, Szu, Cohen;

Properties of $W(x,\xi)$

Properties:
- W is always real
- W can be positive and negative (pseudo-density)
- W is in general positive near the ray manifold with (quantum) small-scale oscillations
- W contains the full information about u(t)
- W has no preferred direction or coordinates

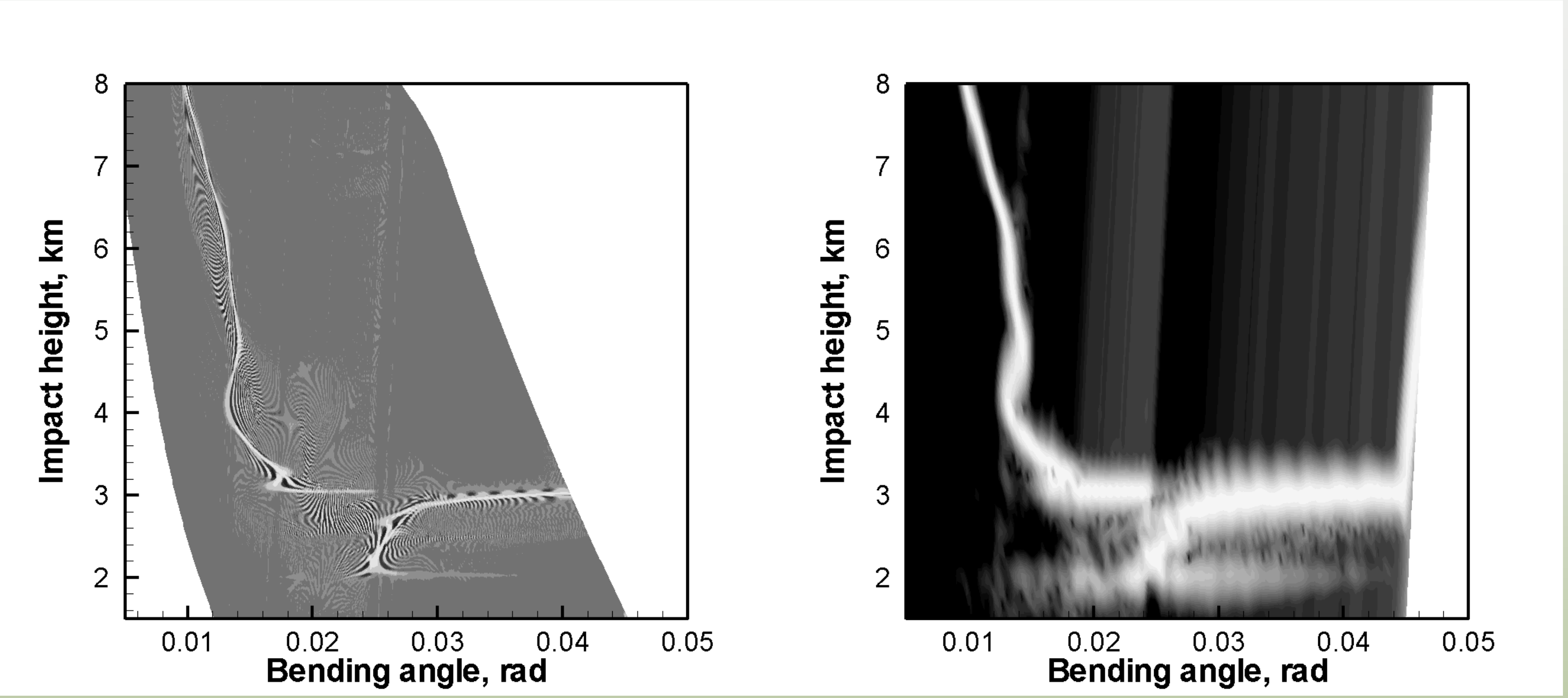
Consequences:
- W is a global integral transform => <u>high resolution</u>
- W can identify multipath in the presence of <u>strong horizontal gradients</u>

WOP simulation 1

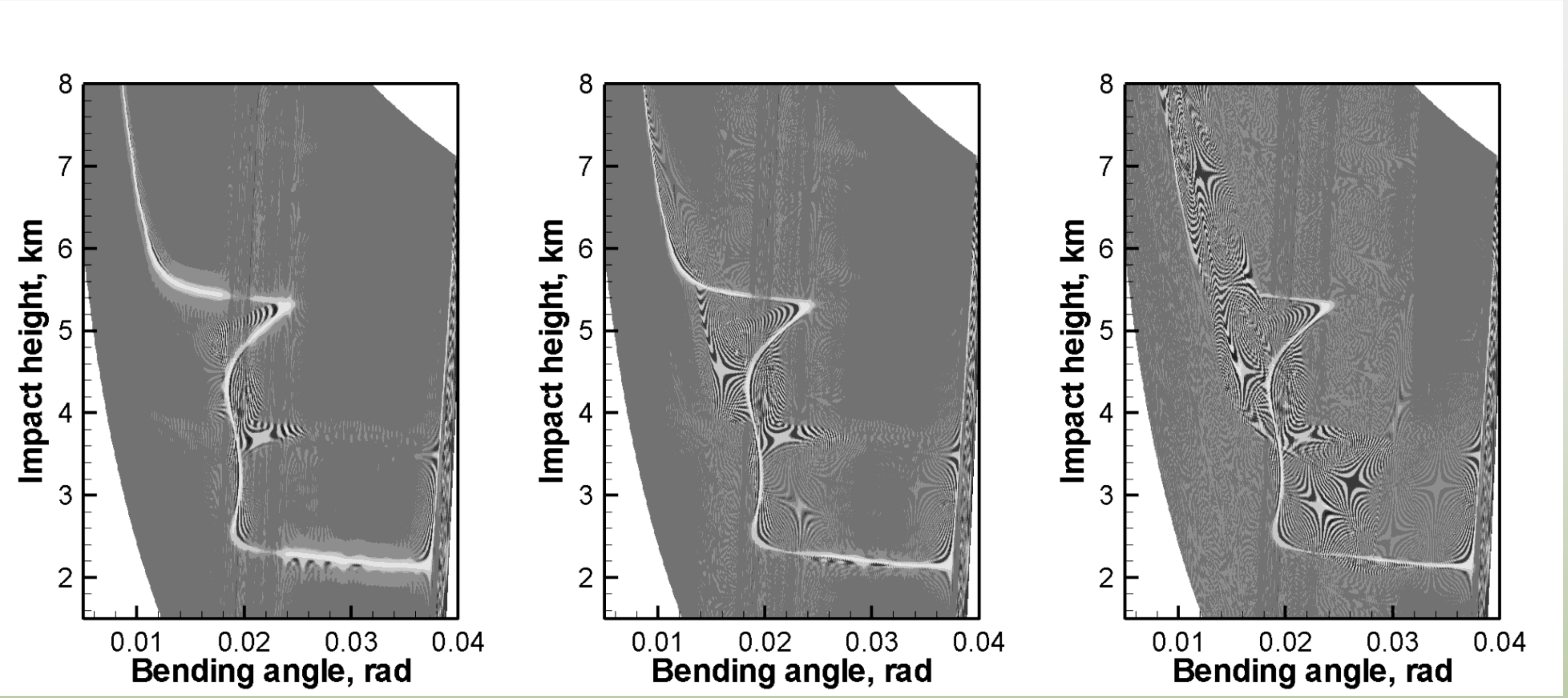


WOP: Wave Optics Simulation

WOP simulation 2

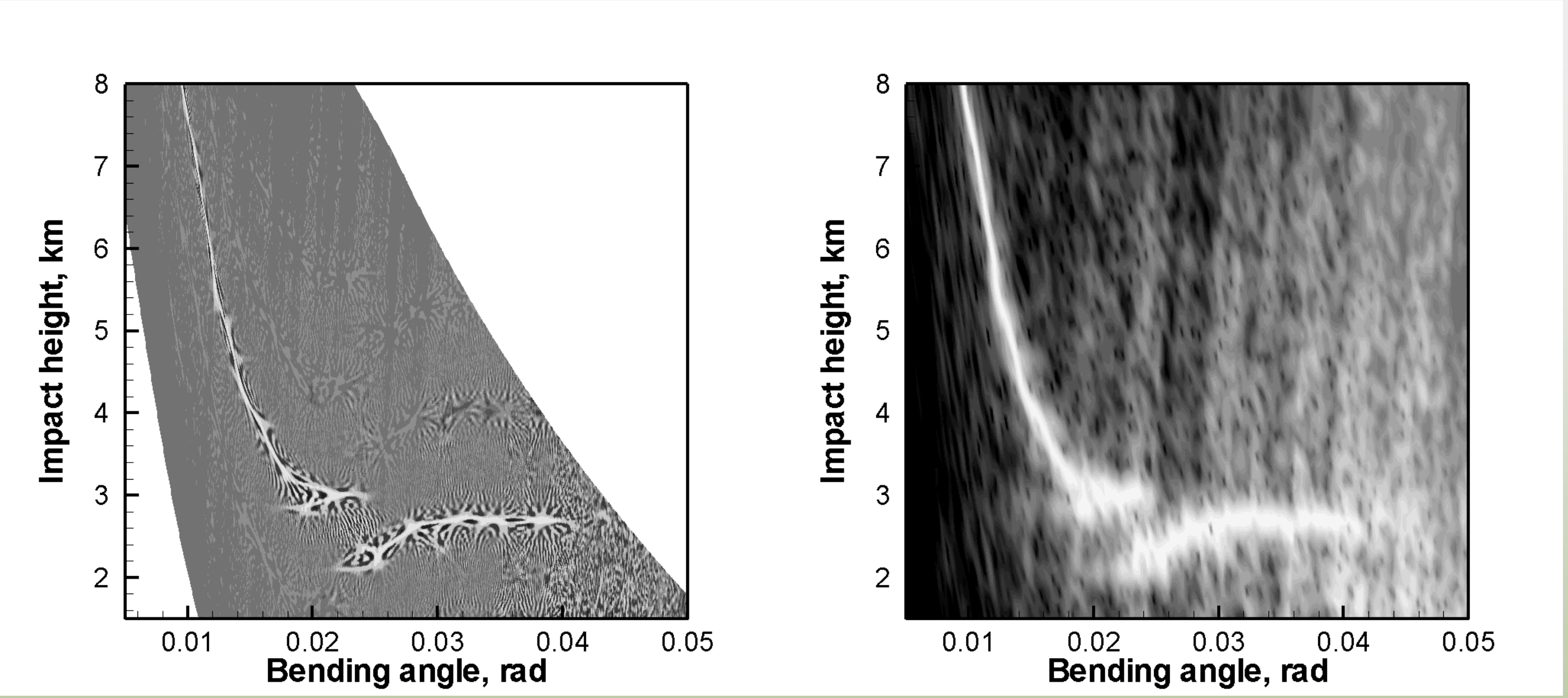


WOP simulation 3: Windowed W

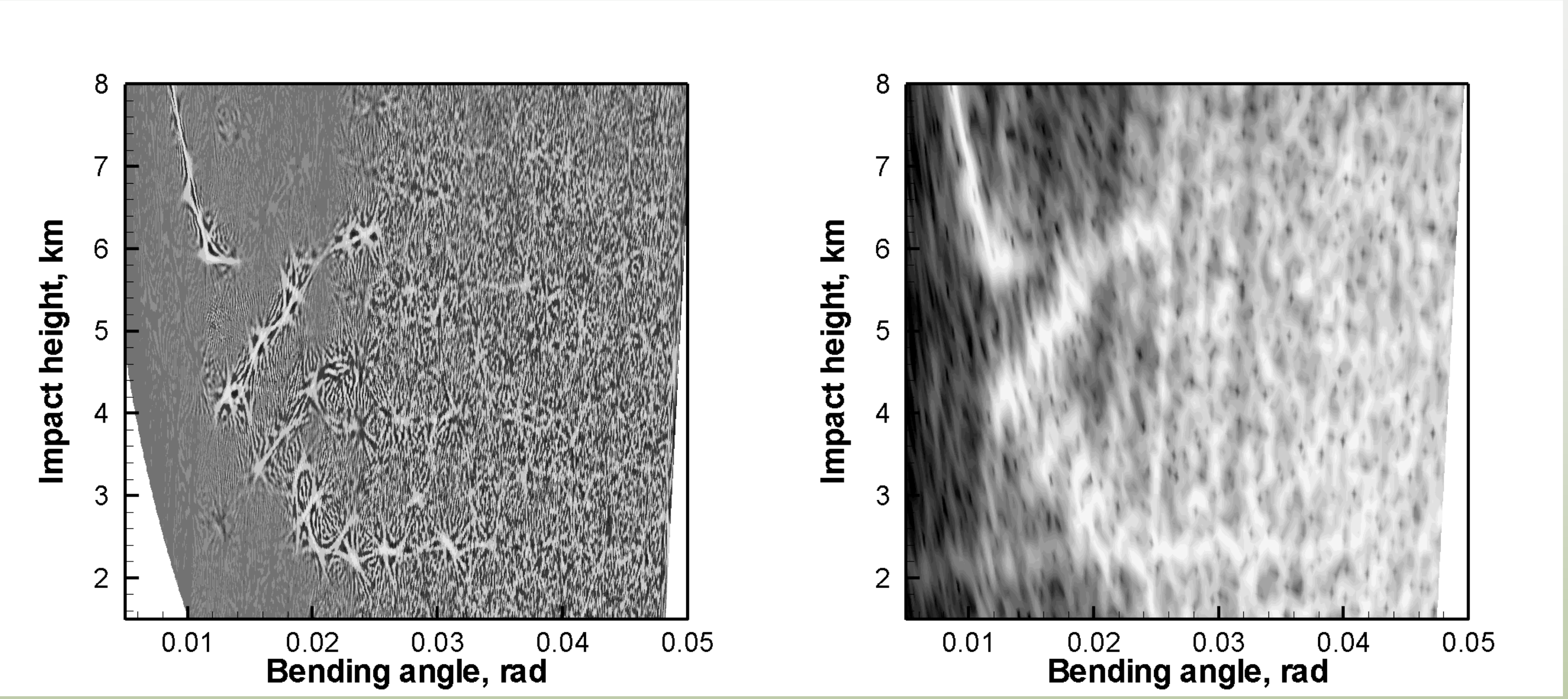


-> Different convolutions with Gaussian weights in order to reduce quantum oscillations (right: W)

COSMIC case 1 (non-tropical)



COSMIC case 2 (tropical)



WDF Retrieval

Bending angle

$$\varepsilon(p) = \int \varepsilon' \, w(\varepsilon', p) \, d\varepsilon' / A(p)$$

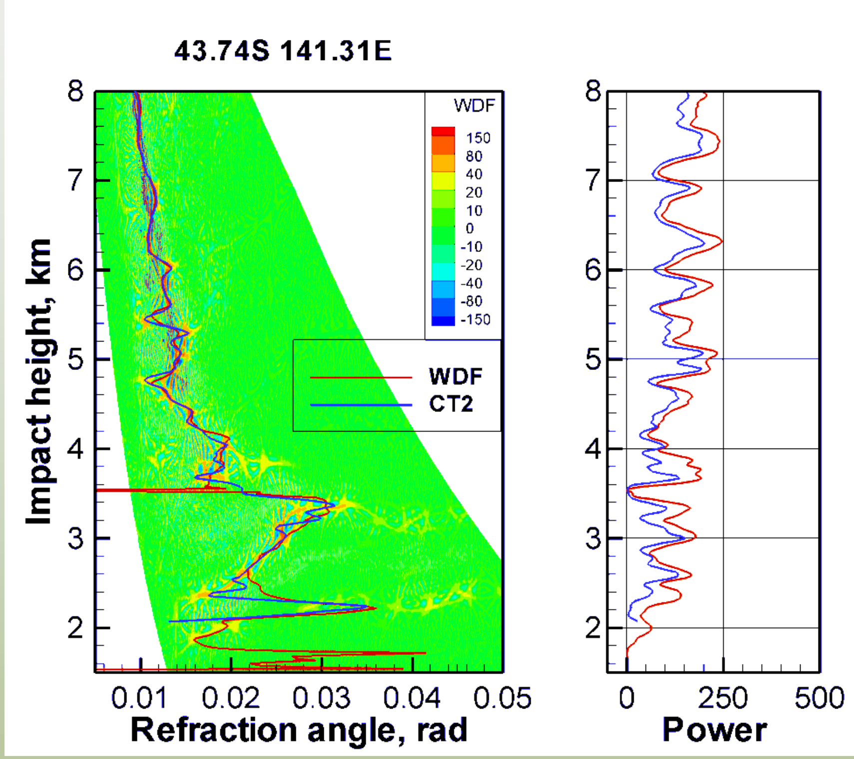
Amplitude

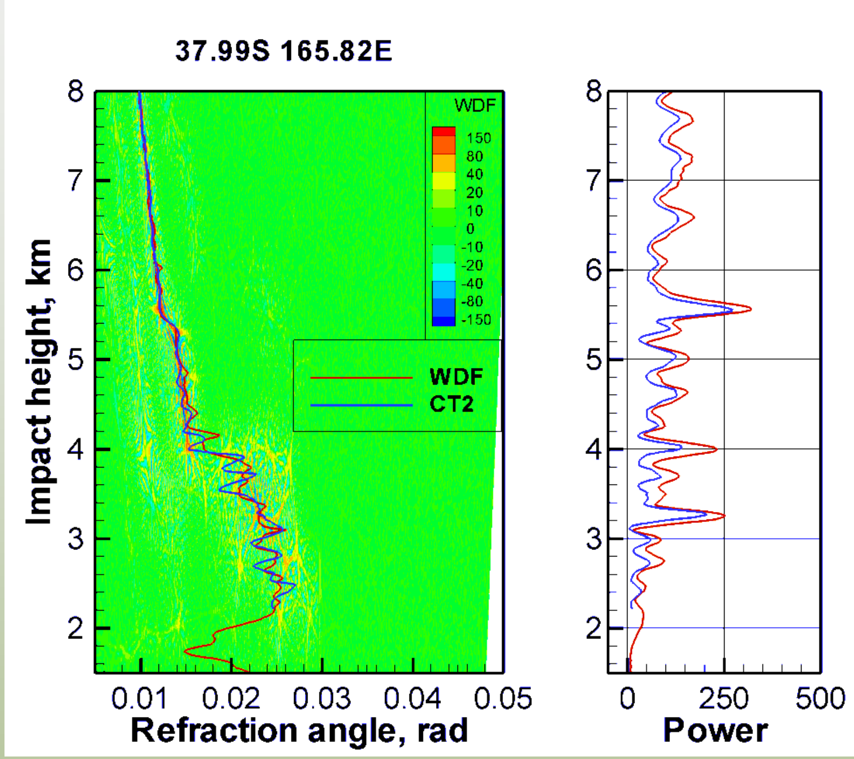
$$A(p) = \int w(\varepsilon', p) \, d\varepsilon'$$

where $w(\varepsilon', p)$ is the WDF spectrum.

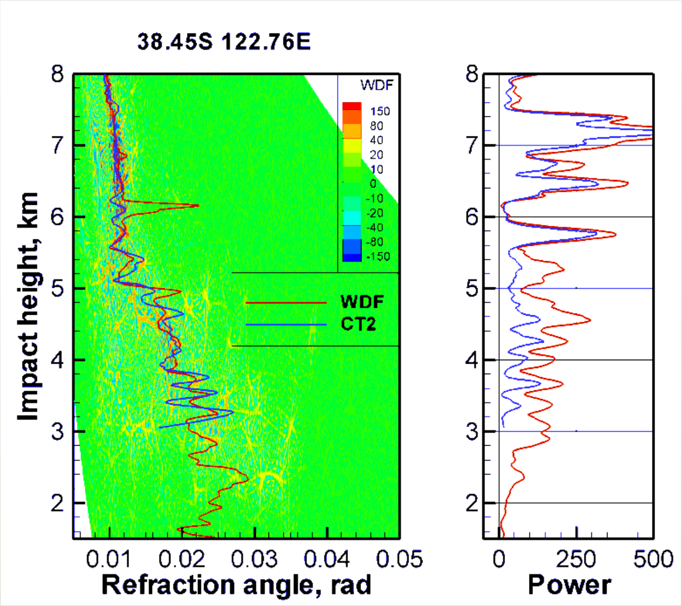
Reference: Sokolovskiy, Radio Science (2001)

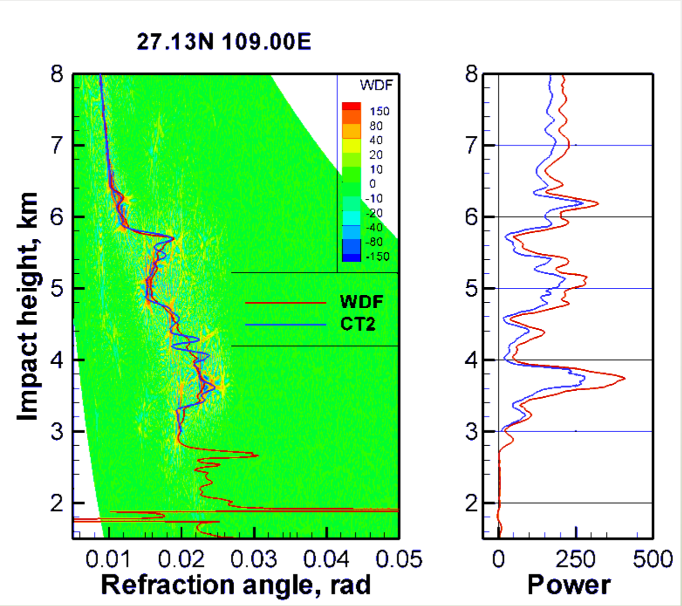
COSMIC (1 Jan 2007)

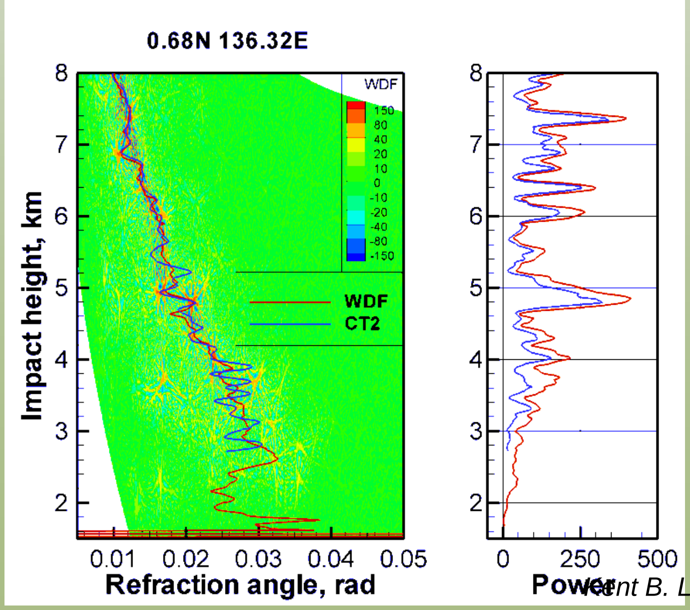


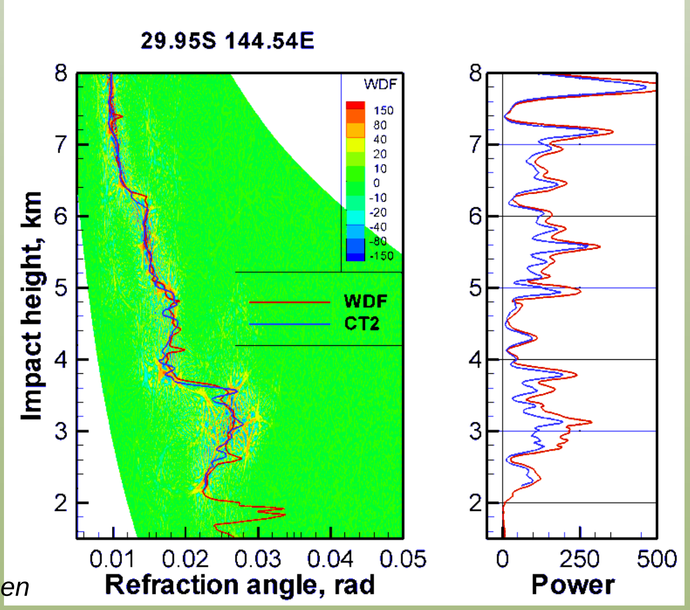


COSMIC (1 Jan 2007)

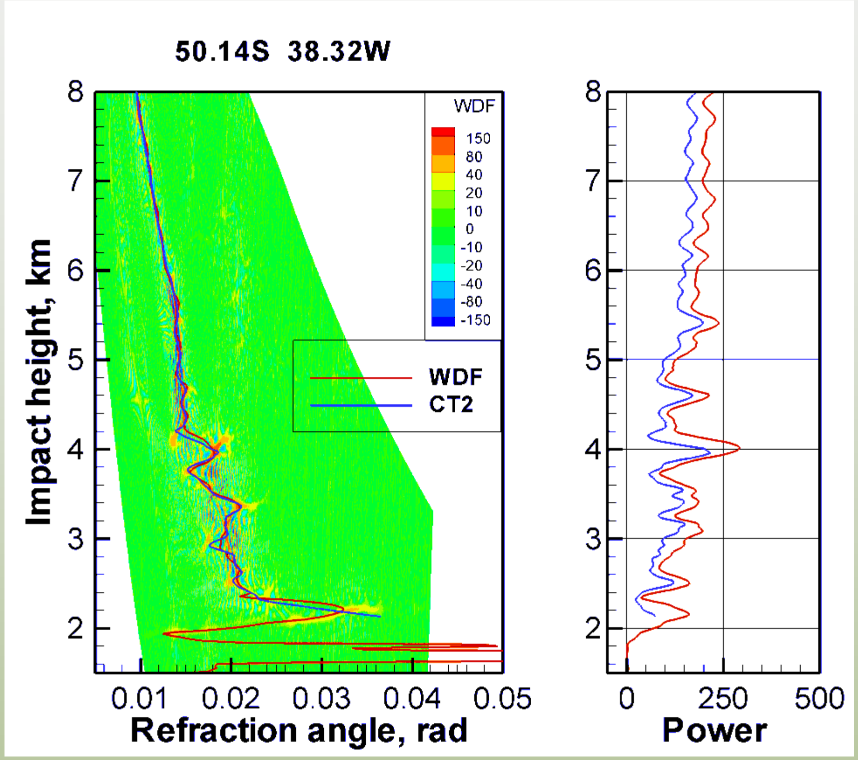


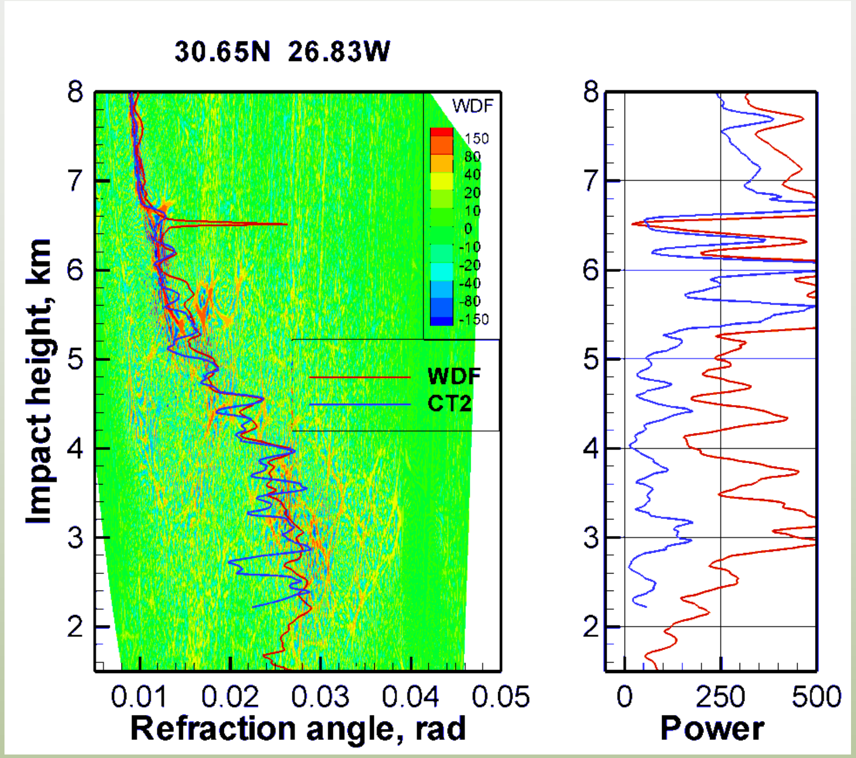




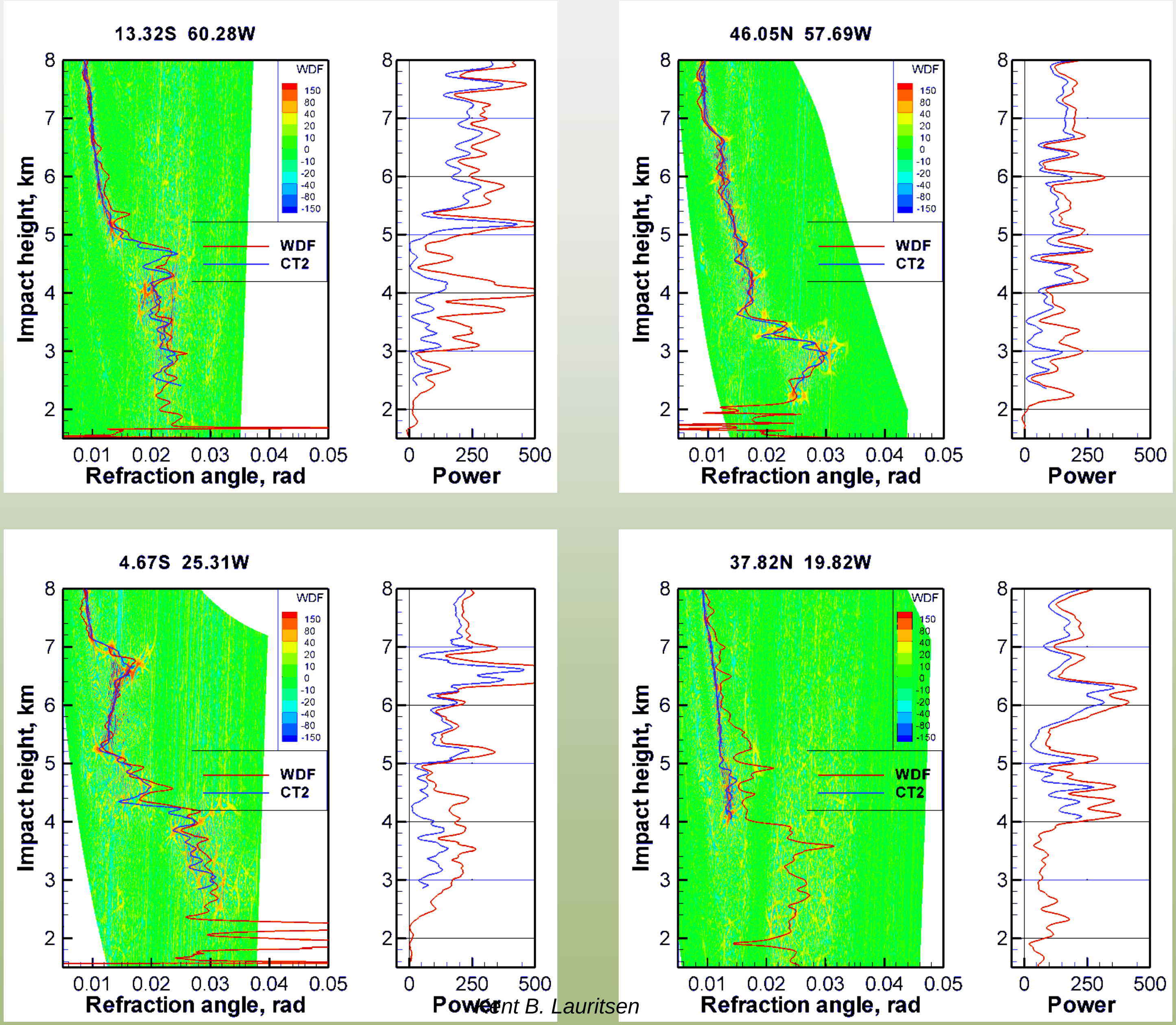


GRAS (30 Sept 2007)





GRAS (30 Sept 2007)



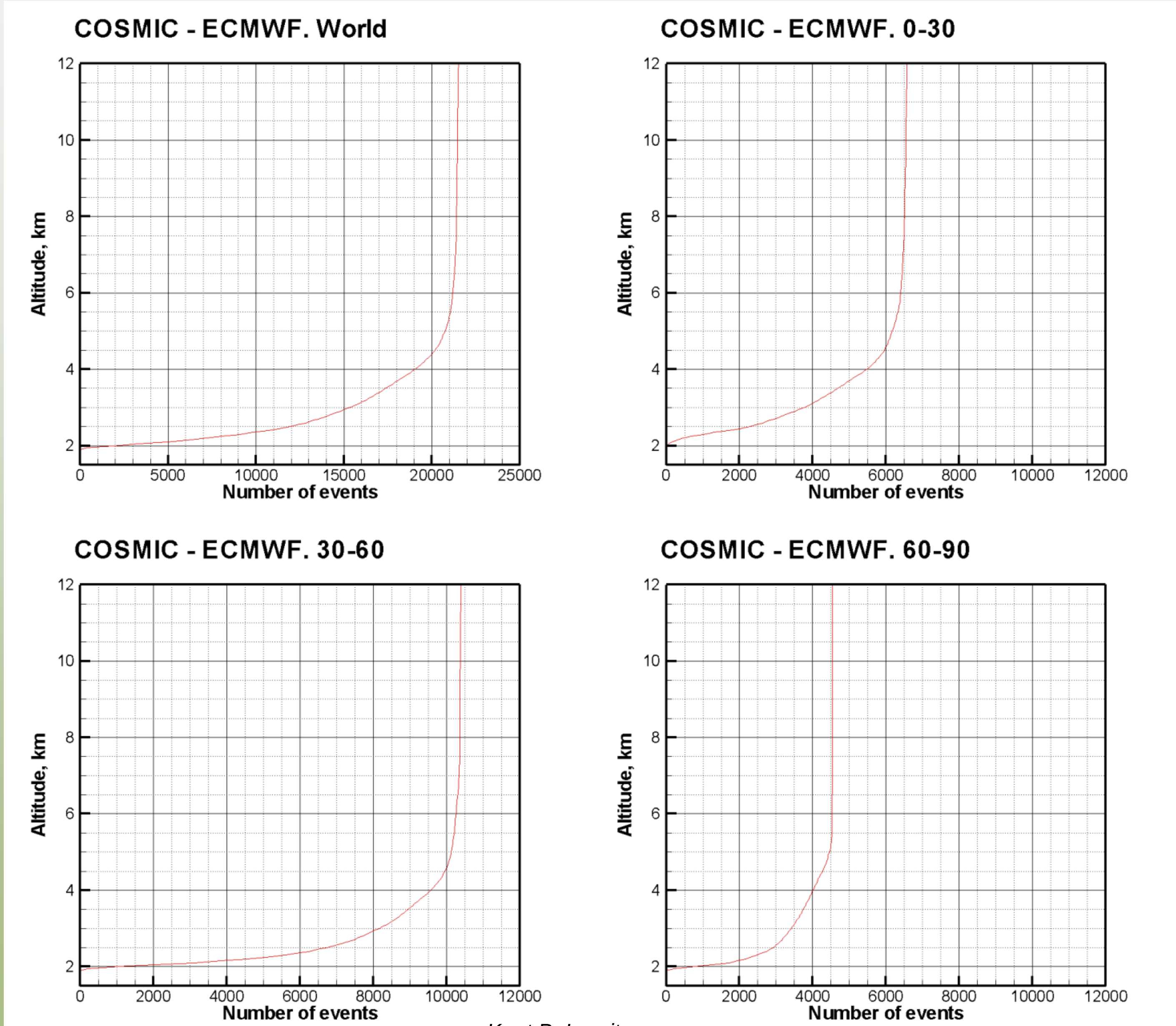
Statistics

The figures show O-B/B statistics for bending angle & refractivity

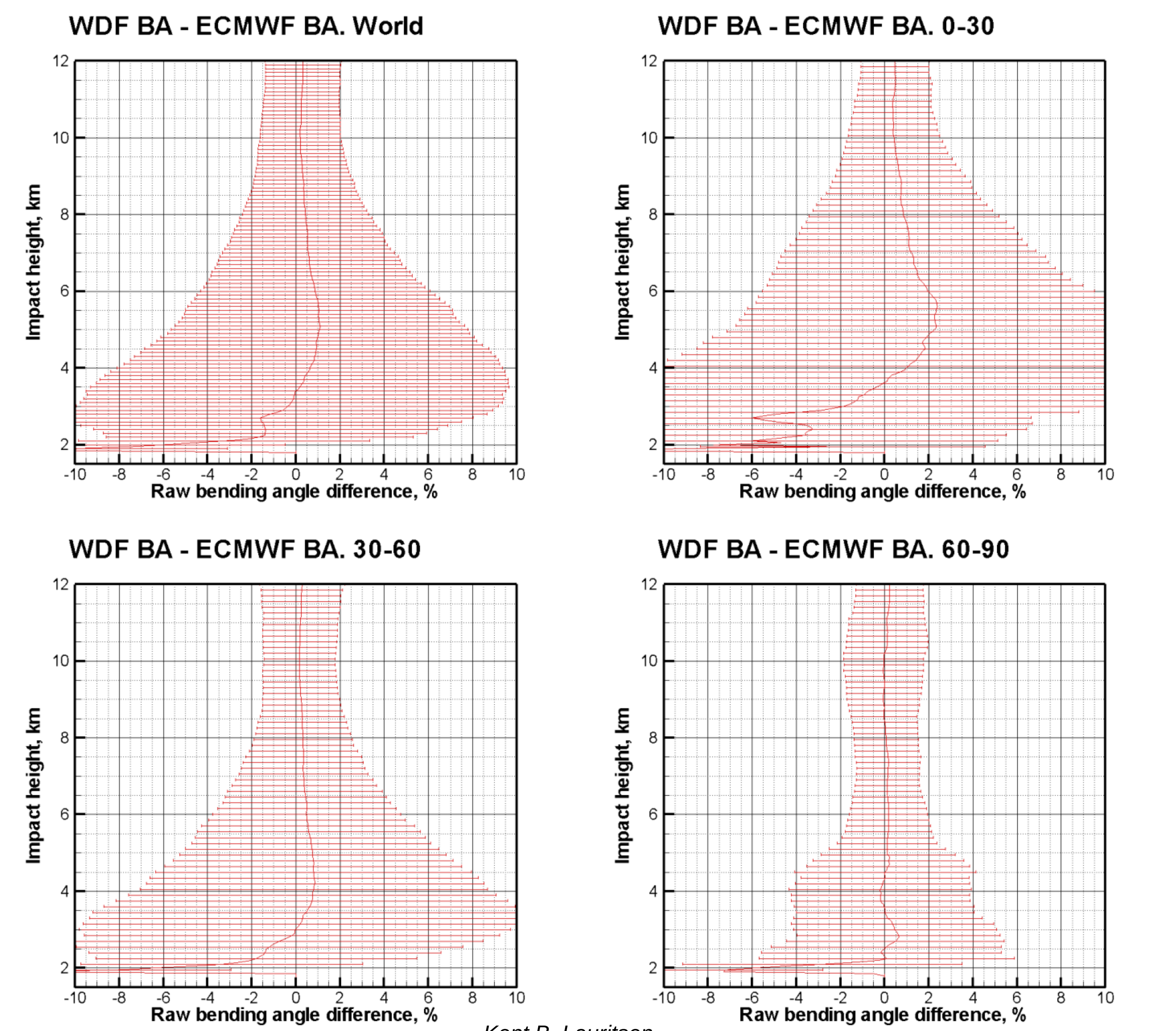
WDF: Wigner retrieval

CT2: CT2 wave optics retrieval

WDF and CT2 for COSMIC data



COSMIC: WDF BA



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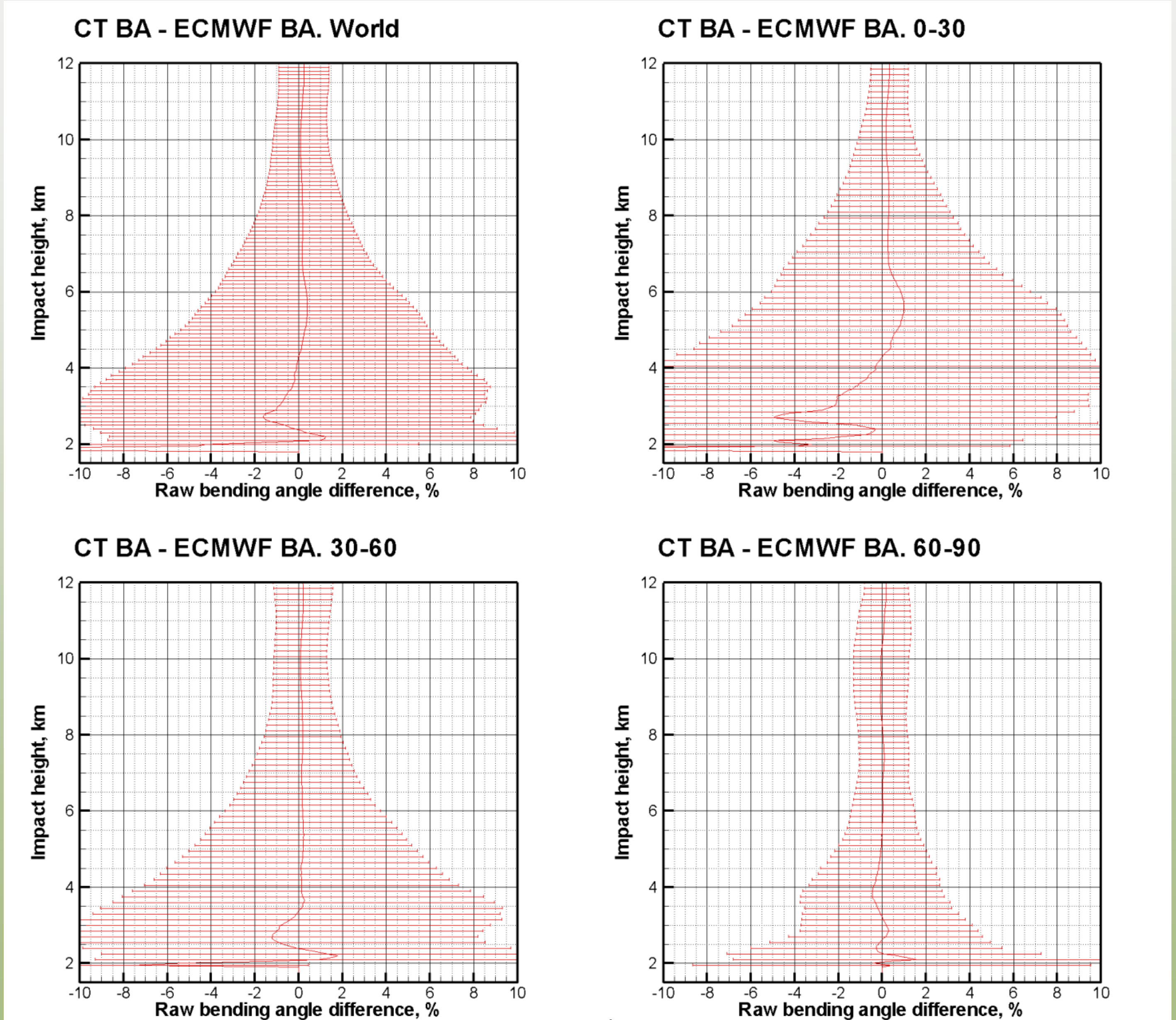
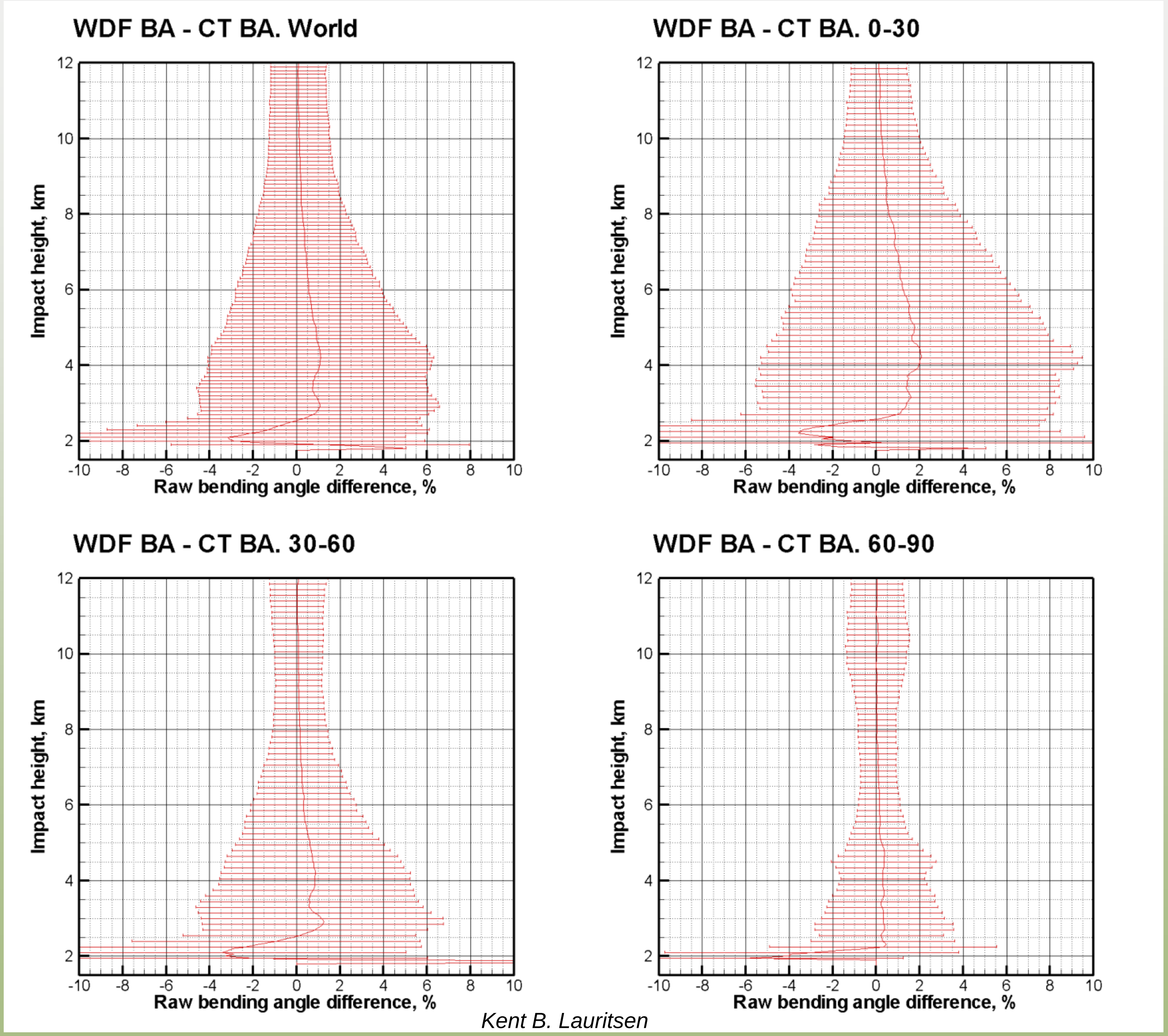
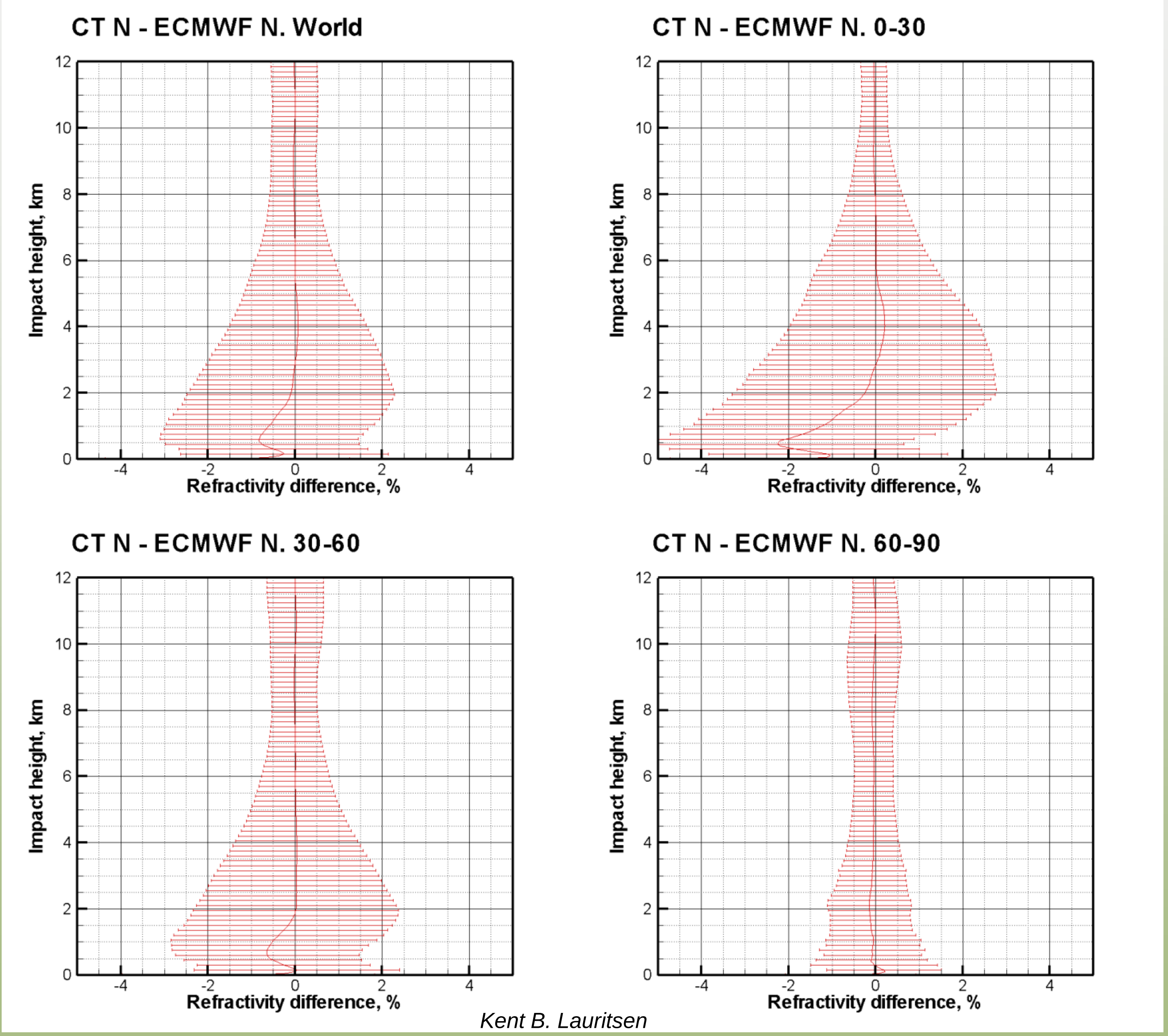
COSMIC: CT2 BA



COSMIC: WDF and CT2 BA

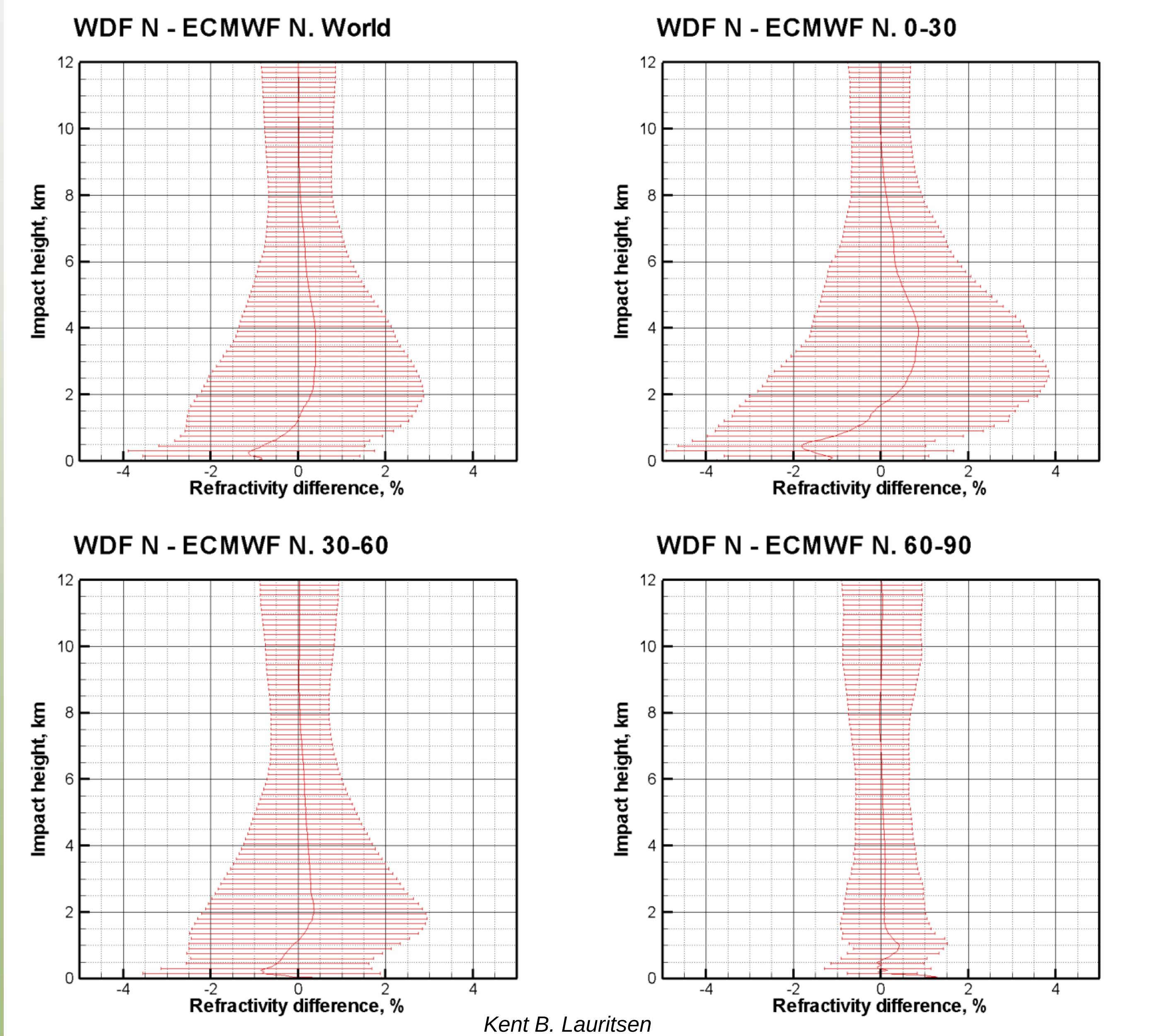


COSMIC: CT2 N



CT N - ECMWF N. World
CT N - ECMWF N. 0-30
CT N - ECMWF N. 30-60
CT N - ECMWF N. 60-90
Impact height, km
Refractivity difference, %
0
2
4
6
8
10
12
-4
-2

COSMIC: WDF N



WDF N - ECMWF N. World
WDF N - ECMWF N. 0-30
WDF N - ECMWF N. 30-60
WDF N - ECMWF N. 60-90
Impact height, km
Refractivity difference, %

Conclusions

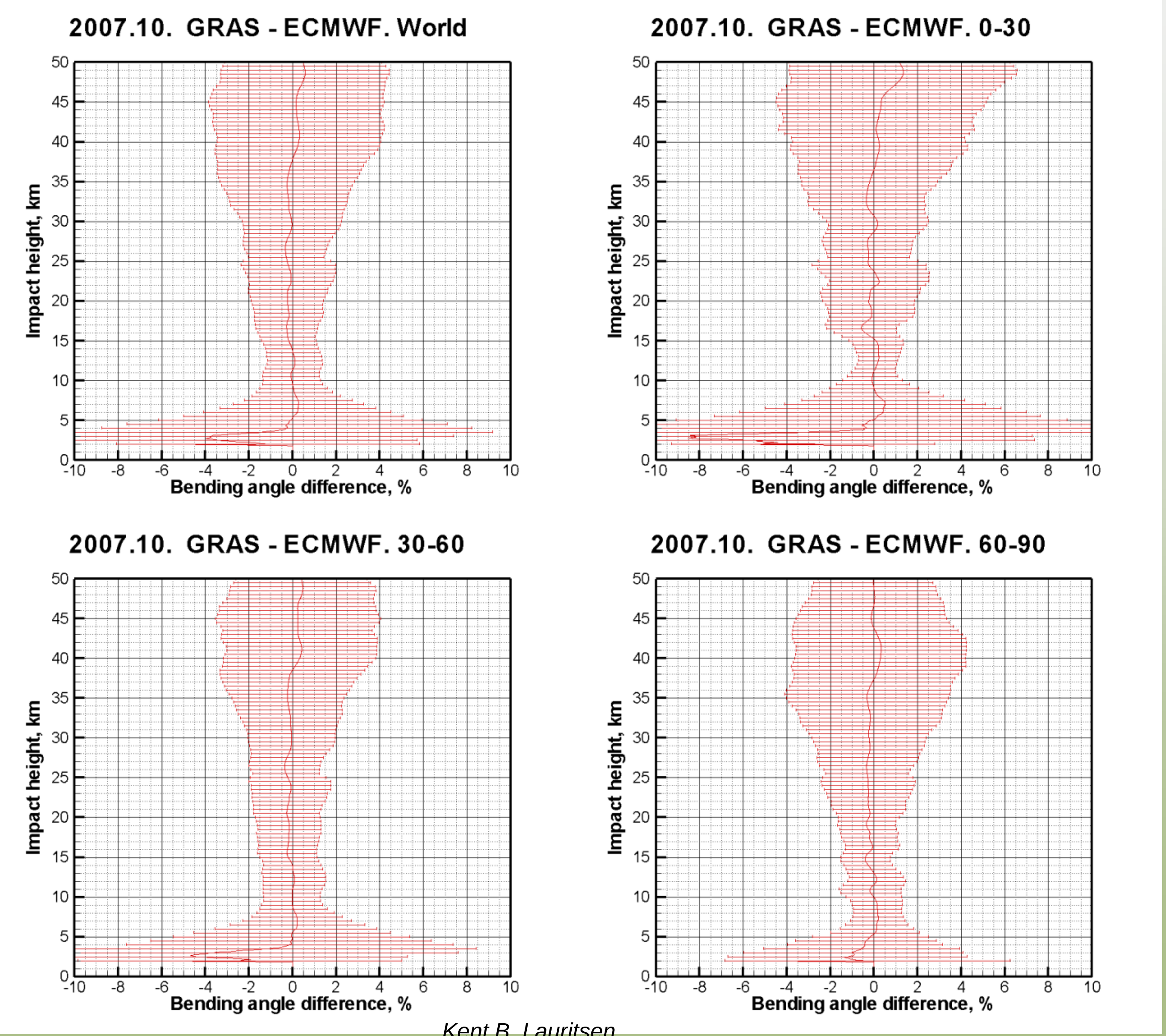
- The Wigner distribution function (WDF) maps a 1D wave function to a 2D time-frequency representation in phase space similar to a radio holographic analysis.

- Examples and simulations for COSMIC and GRAS RO data show that WDF allows for a sharp localization of the details of bending angle profiles.

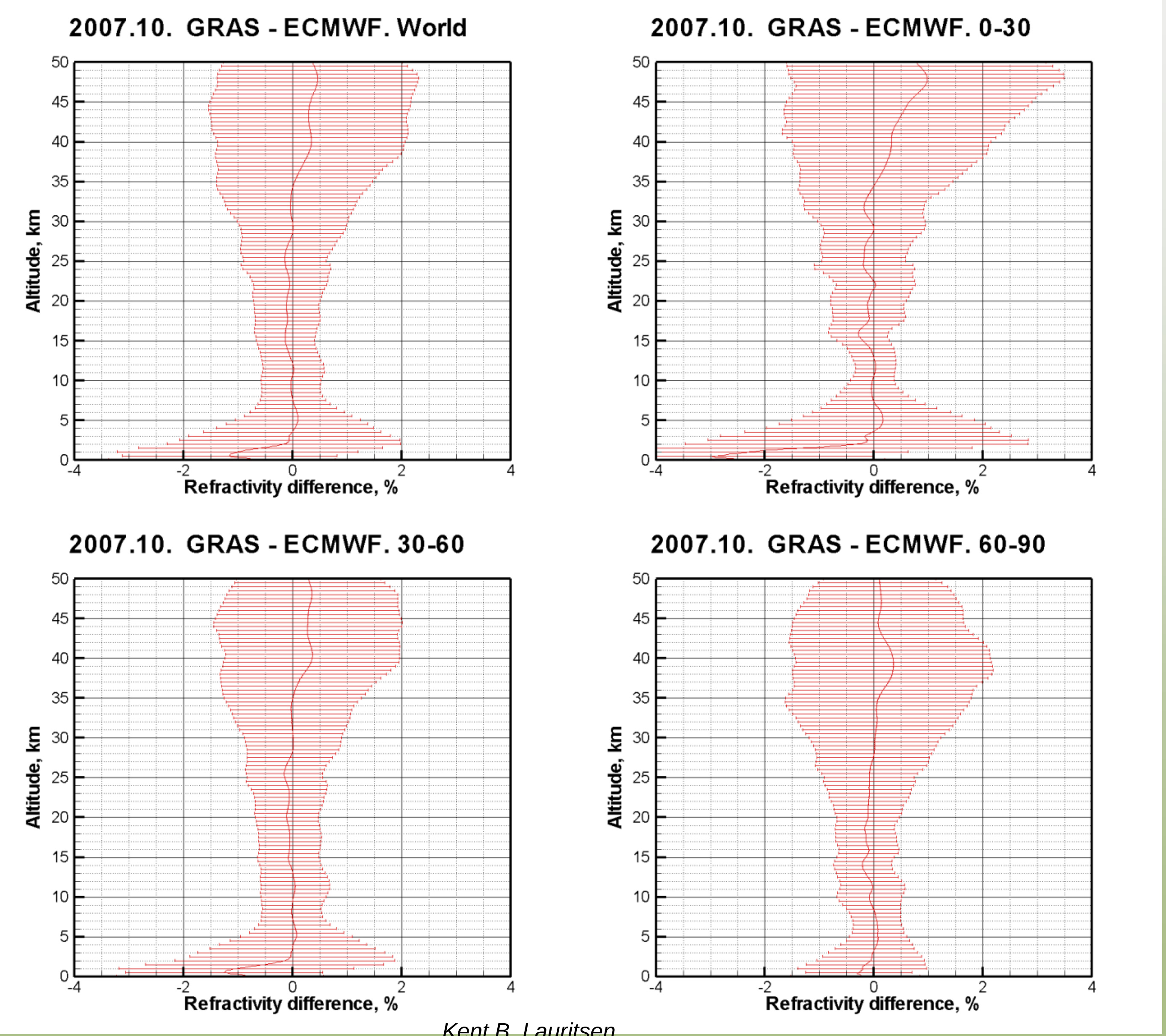
- Statistical analysis of bending angles and refractivities obtained by WDF and CT2 are in agreement.

Fin

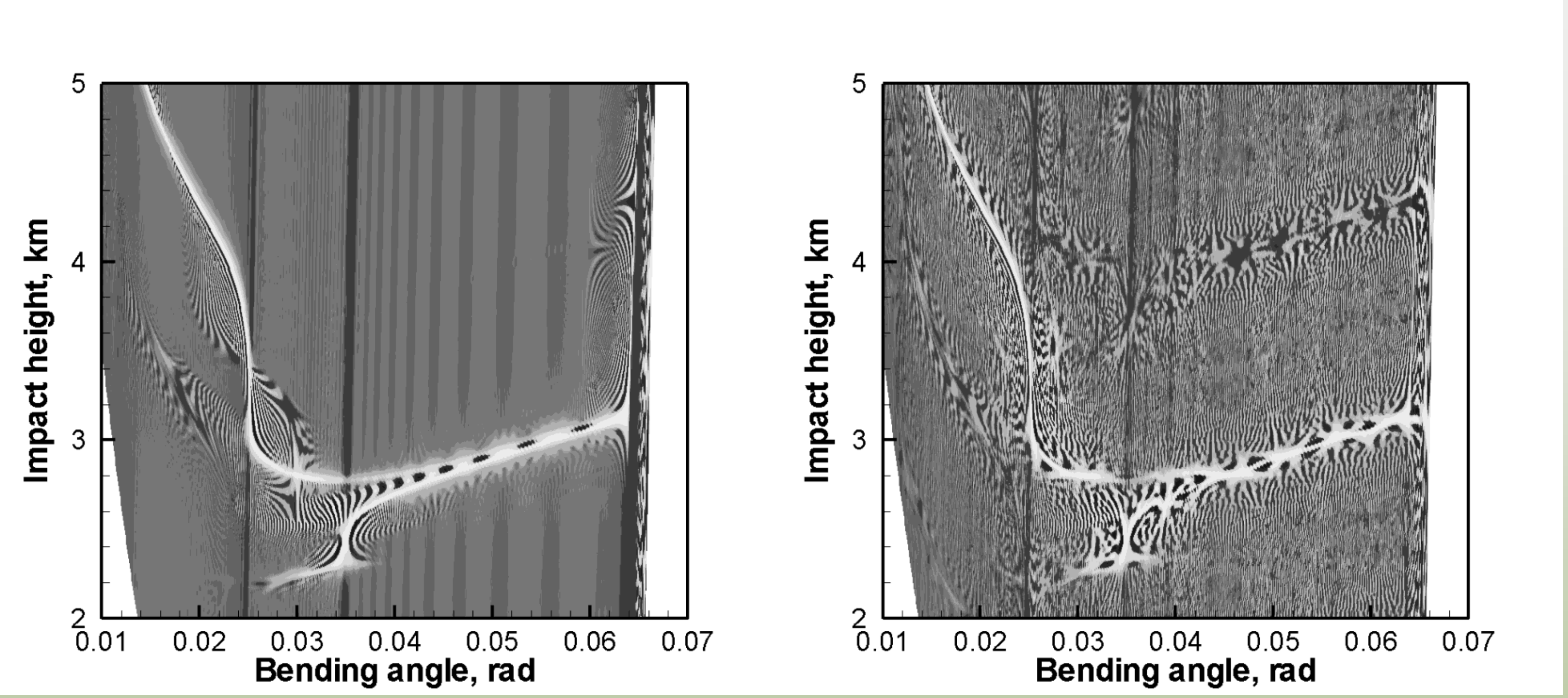
GRAS RS data: Oct 2007 smooth BA



GRAS RS data: Oct 2007 REF



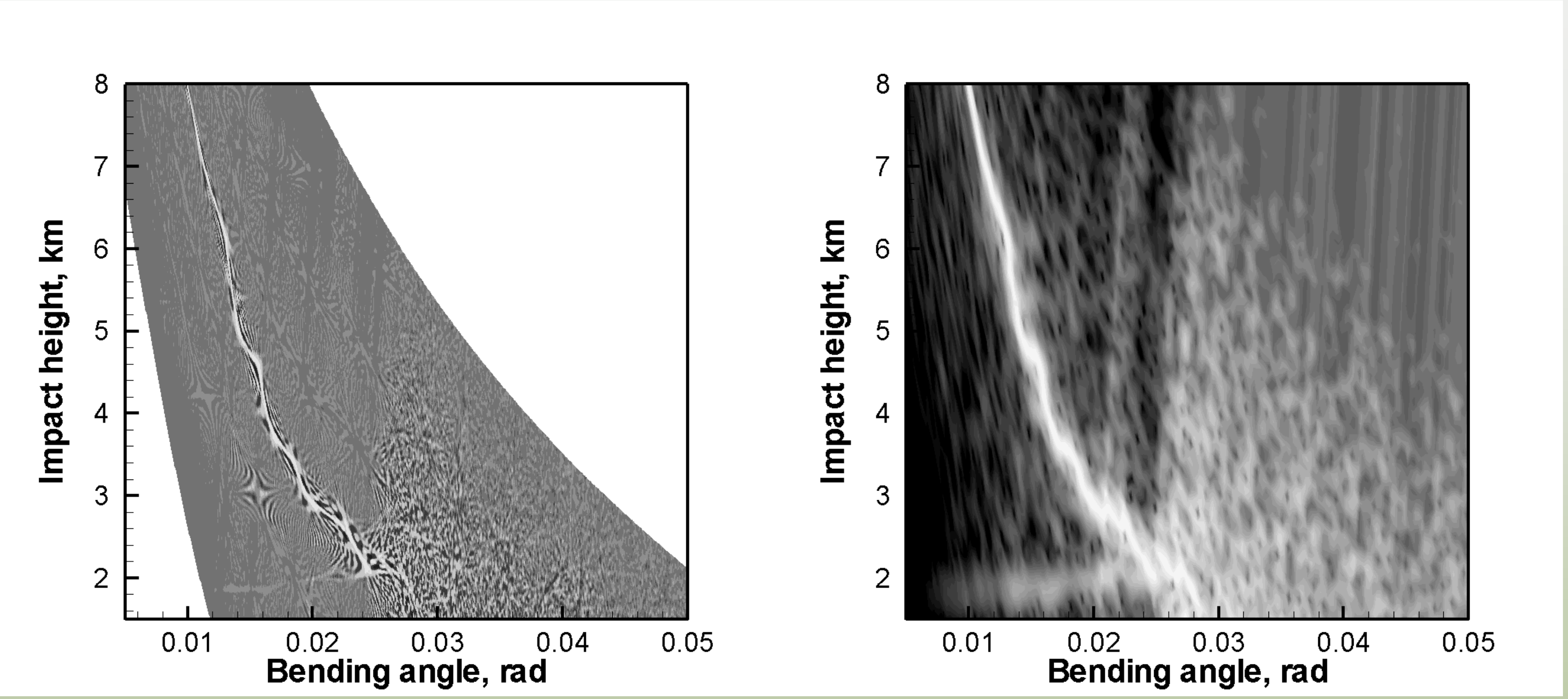
WOP simulation 4



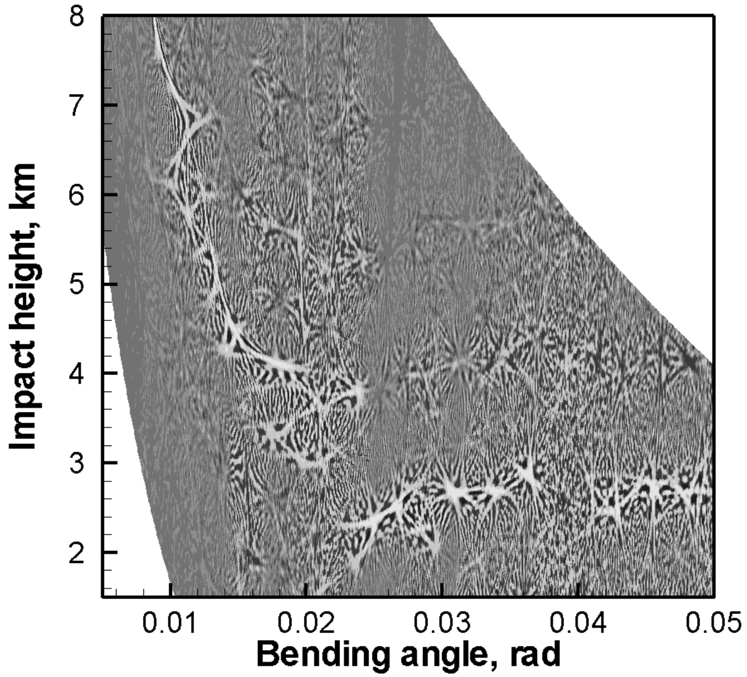
left: strong horizontal gradients

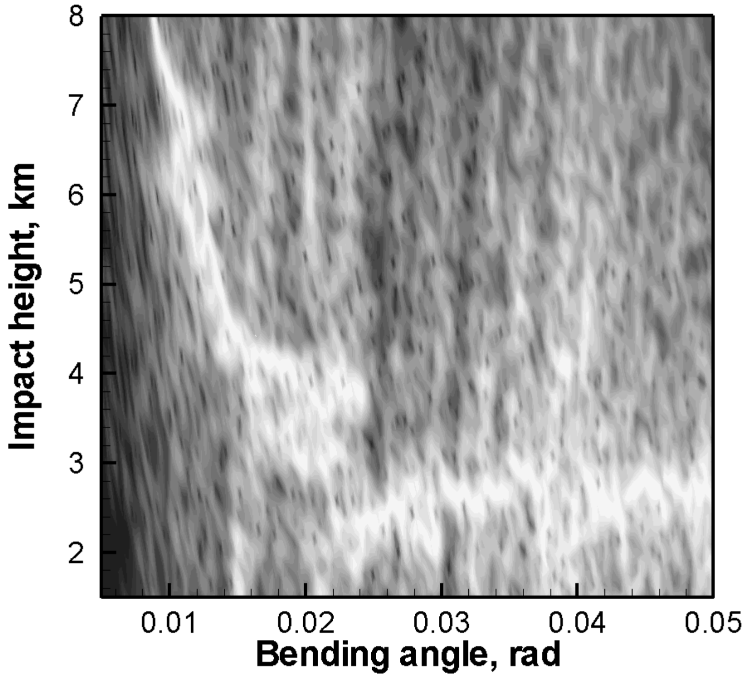
right: realistic model for turbulent fluctuations added

COSMIC case 3 (tropical)

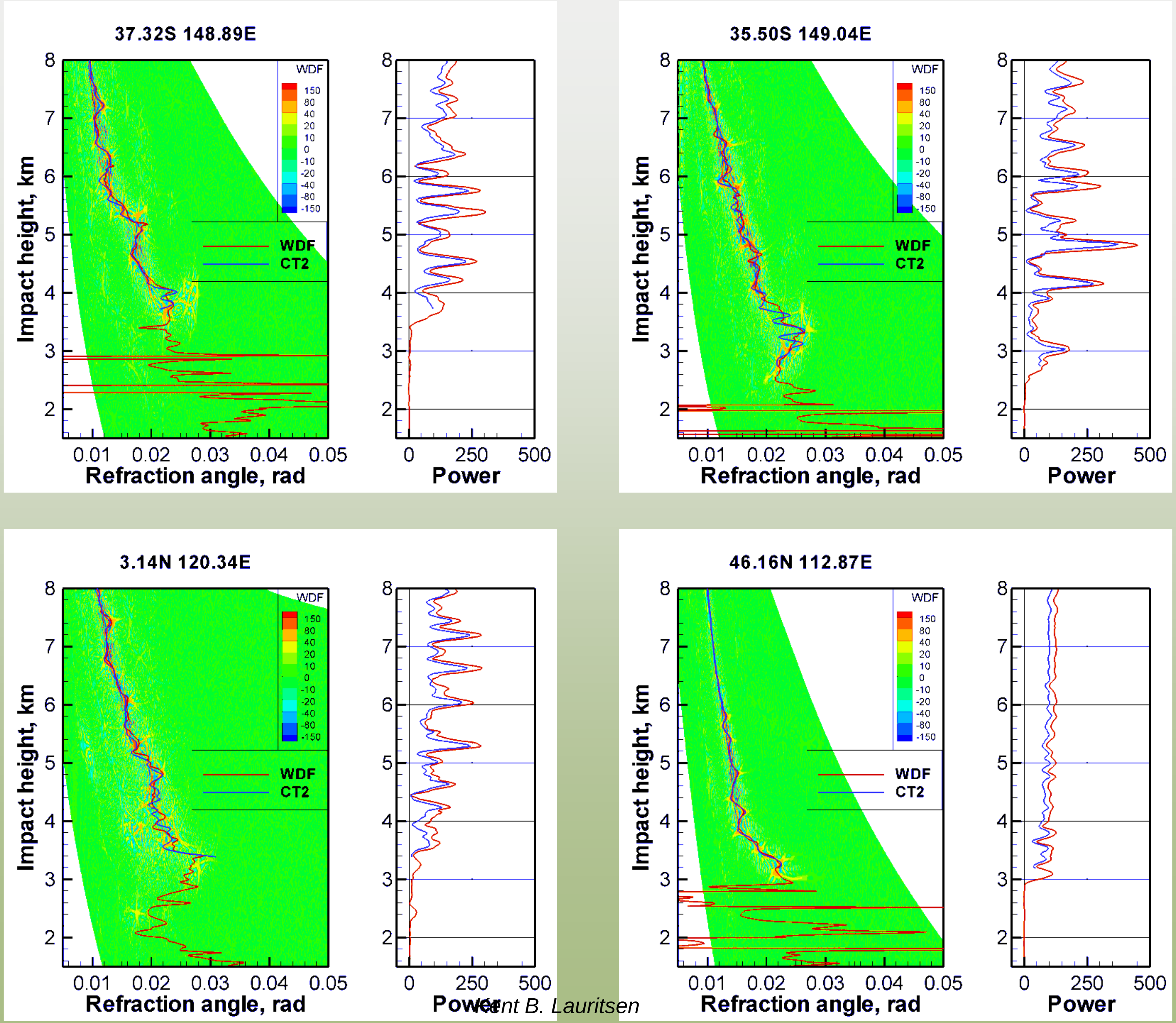


COSMIC case 4 (tropical)





COSMIC (1 Jan 2007)



GRAS (30 Sept 2007)