Investigating severe troposphere weather effects on GNSS signal paths using numerical ray tracing

Robert Norman, John Le Marshall, Witold Rohm, Brett Carter, Yubin Yuan and Kefei Zhang

SPACE Research Centre
School of Mathematical and Geospatial Sciences
RMIT University, Australia
www.rmit.edu.au/space
Outline

• 3-D numerical ray tracing technique
• 3-D analytic ray tracing technique
• Simulated GNSS radio occultation (RO) results
• Severe troposphere weather event
• Simulated GNSS RO signal paths traversing the severe weather event
Numerical ray tracing
Generally requires a form of Haselgrove equations where these equations are integrated at each step along the ray path.

Analytic ray tracing
Analytic ray tracing (ART) as its name suggests uses explicit equations to represent the ionosphere and lower atmosphere as well as the ray parameters. ART simulates ray paths much quicker than numerical ray tracing techniques, but in the past has been restricted to much simpler spherically stratified ionospheric models.

We have developed the new 3-D Segment Method Analytical Ray Tracing (3D-SMART) technique.
Refractive index in the Ionosphere

The Appleton-Hartree equation which is sometimes referred to as the Appleton-Lassen equation describes the phase refractive index in the ionosphere.

\[
n^2 = 1 - \frac{iZ}{1 - iZ} - \frac{\frac{1}{2}Y^2 \sin^2 \theta}{1 - X - iZ} \pm \frac{1}{1 - X - iZ} \left( \frac{1}{4}Y^4 \sin^4 \theta + Y^2 \cos^2 \theta (1 - X - iZ)^2 \right)^{1/2}
\]

- \( n \) is the complex refractive index in the ionosphere
- \( X \) is proportional to electron density
- \( Y \) is proportional to magnetic field (\( y=0 \), No magnetic field (NF))
- \( Z \) is proportional to collision frequency

'+' sign represents the Ordinary mode, and
'-' sign represents the Extraordinary mode.

* Collision frequency ignored i.e., \( Z=0 \). Dealing with Real refractive index.
Numerical Ray Tracing Program

- Traces ray tubes from GNSS to LEO satellites
- Our ray tube technique involves integrating 18 differential equations simultaneously at each step along the ray path
- International Reference Ionosphere (IRI) model
- Can trace ordinary (O), extraordinary (X) as well as the no field (NF) ray paths
- Program has 2 choices for the magnetic field model
  - Earth centred dipole magnetic field model
  - POGO 68/10 magnetic field Legendre model (IRI)
- Homing-In capability
- Able to trace ray paths and determine group path, phase path, ground range, perigee height, transmitted and received elevation and azimuth angles as well as the divergent signal strength.
Analytic ray tracing - QCS model

\[ y = f_N^2 = \frac{A}{r^3} + \frac{B}{r^2} + \frac{C}{r} + D \]

Where \( y, y', y'' \) are all smooth and continuous and using the method of least squares.
QCS ray parameters

\[ P = \int \frac{r^2 \mu^2 \, dr}{r \sqrt{r^2 \mu^2 - r_o^2 \cos^2 \beta_o}} \]

\[ r^2 \mu^2 - r_o^2 \cos^2 \beta_o = ar^2 + br + c + d / r \]

\[ a = 1 - \frac{D}{f^2} \]

\[ b = -\frac{C}{f^2} \]

\[ c = -\frac{B}{f^2} - r_o^2 \cos^2 \beta_o \]

\[ d = -\frac{A}{f^2} \]

\[ P = \int \frac{(1 - D / f^2) r^2 \, dr}{\sqrt{ar^4 + br^3 + cr^2 + dr}} - \int \frac{C / f^2 r \, dr}{\sqrt{ar^4 + br^3 + cr^2 + dr}} - \int \frac{B / f^2 \, dr}{\sqrt{ar^4 + br^3 + cr^2 + dr}} - \int \frac{A / f^2 \, dr}{r \sqrt{ar^4 + br^3 + cr^2 + dr}} \]

\[ P_{\phi_i} = -\frac{1}{2f^2} \frac{\partial f_N^2}{\partial \phi} P'_i \]
Year: 2000
Day of year: 267
Local time: 1200 Hours
Transmitter Geomagnetic Latitude: 52.02 S
Transmitter Geomagnetic Longitude: 225.3 E
Propagation direction: Southward
Operating frequency of propagation: 12.0 MHz
GPS L-Band Propagation

Height versus Refractivity for GPS L1 and L2 frequencies using IRI-2007, with input longitude 225.28º E, latitude 11.85º N, year 2010, day 267 and 1200LT.

GPS satellite located at height 20,200 km, -50.0º N, 225.28º E. Northward propagation.
March 6th 2010 storm in Melbourne
Storm

Location of the stations considered in this study: SA45, WHIT and MOBS (triangles) plotted over the map of South-Eastern Australia (Victoria and South Australia). Open circles shows radar-inferred locations of most severe storm cells, the arrow points to the direction of severe event propagation.

Evolution of the refractivity over time at 3 Australian GPS stations. MOBS and WHIT are located in Melbourne region whereas SA45 is placed close to Port Augusta. Vertical lines mark the time interval of the cold front passage associated with the storms and intense precipitation.

No storm: $N_o = 320$ N-units, $H = 7.3$ km
Storm: $N_o = 350$ N-units, $H = 6.7$ km, Transv grad = 4 N-units/km

$$N = N_o e^{-\frac{h}{H}}$$
Simulation results
Storm excess phase

Perigee height [km]
Distance [km]

Perigee height [km]
Distance [km]

Perigee height [km]
Storm excess phase [m]
Summary and Conclusions

- We have developed a 3-D numerical ray tracing technique that can simulate the GNSS RO signal paths. The technique traces a ray tube and can home in to the desired receiver location.
- We have also developed a new 3-D analytic ray tracing technique.
- The 3-D numerical ray tracing technique can be used to determine the lateral displacements in the GNSS signal paths.
- In the example chosen featured a large transverse horizontal gradient of 4 N-units/km near the Earth’s surface and resulted in an excess phase delay of ~1.7 m for ray paths having a perigee/tangent height of 1.5 km.